

and $\Delta_0(k_F) = 1.2kT_c$. The energy gap is nodeless; this is expected for the lowest energy state whenever a nodeless solution of the gap equation exists.⁵ An important consequence of the nodeless gap is that the single-particle contribution to neutron-matter heat capacity contains the exponentially vanishing factor $e^{-T_c/T}$, just as for an isotropic superfluid.

The rotational degeneracy associated with the arbitrariness of the $\theta = 0$ direction is resolved in a neutron star by considerations of its stability. The lowest-energy configuration for the star is achieved when the gap polar axis ($\theta = 0$), along which the compressibility is greatest, orients itself in the radial direction in the star. There will, of course, exist some collective excitation modes⁵ in the star in which the gap polar axis departs from its locally preferred direction.

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EVIDENCE FOR Ξ RESONANCES IN THE $\Xi(1530)\pi$ SYSTEM*

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Evidence is presented for the decay of the $\Xi(1820)$ and $\Xi(1930)$ resonances to $\Xi(1530)\pi$, and for an $I = \frac{1}{2}$ assignment to these states. By combining data from the $\Xi\pi$ and $\Xi(1530)\pi$ decays of these resonances, we have determined their masses (1820 ± 7 , 1963 ± 8) MeV, widths (64 ± 23 , 89 ± 33) MeV, and branching ratios [$\Xi\pi/\Xi(1530)\pi = 1.5^{+0.6}_{-0.4}$, $2.8^{+0.7}_{-0.6}$].

The existence of Ξ resonant states in the $\Xi(1530)\pi$ system has yet to be convincingly demonstrated.¹⁻⁴ In this paper, evidence is presented for the decay of $\Xi(1820)$ and $\Xi(1930)$ into $\Xi(1530)\pi$ channels. The data have been analyzed to determine mass, width, and isospin values

for these resonances. In addition, a comparison with our previously reported result⁵ on resonant states in the $\Xi\pi$ system yields branching-ratio information for these particles.

The data are based on the analysis of 10^6 pictures of K^-p interactions at 2.87 GeV/c momen-

tum taken in the Brookhaven National Laboratory 31-in. bubble chamber. The sample corresponds to 30 events/ μb . All film was scanned twice for events with at least two visible decays of strange particles. Those events with one or more successful kinematic fits (confidence level $\geq 0.1\%$) were inspected for consistency with observed ionizations.

The channels of interest in this study are

$$K^-p \rightarrow \Xi_1^0 \pi^+ \pi^- K_1^0, \quad 86 \text{ events}; \quad (1)$$

$$K^-p \rightarrow \Xi_1^- \pi^+ \pi^- K^+, \quad 139 \text{ events}; \quad (2)$$

$$K^-p \rightarrow \Xi_1^- \pi^+ \pi^0 K_1^0, \quad 64 \text{ events}; \quad (3)$$

$$K^-p \rightarrow \Xi_2^- \pi^+ \pi^0 K_1^0, \quad 80 \text{ events}. \quad (4)$$

The subscripts 1 and 2 above denote the V^0 being seen or unseen, respectively. The number of events indicate unique fits except for seven events in Reaction (4) which are ambiguous with $\Sigma^- \pi^+ K_2^0 K_1^0$ and have been weighted by $\frac{1}{2}$. A sample of 22 events with fits to both Reaction (1) and the four-constraint fit $\Lambda \pi^+ K^+ K_1^0$ constitutes the only additional ambiguity, and has not been included in the subsequent analysis.⁶

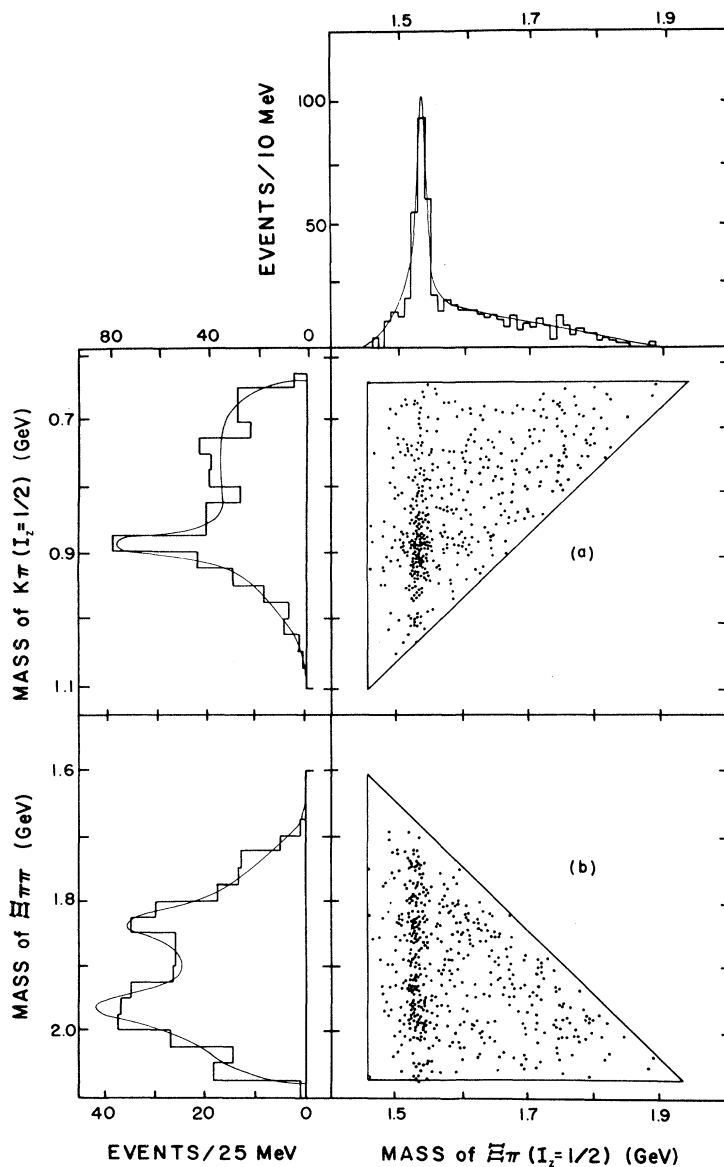


FIG. 1. Triangle plots of (a) $M(K\pi)$ vs $M(\Xi\pi)$ and (b) $M(\Xi\pi\pi)$ vs $M(\Xi\pi)$ for $|I_z| = \frac{1}{2}$ combinations of Reactions (1)-(4). Two combinations are plotted for Reactions (3) and (4). The corresponding mass projections are also shown, with best-fit curves (see text) superimposed.

The main features of the data from Reactions (1)-(4) are shown in Figs. 1(a) and 1(b). The $(\Xi\pi)_{I_Z=1/2}$ vs $(K\pi)_{I_Z=1/2}$ mass distribution of Fig. 1(a) shows a strong $\Xi(1530)$ band and a large accumulation in the $\Xi(1530), K^*(890)$ overlap region.⁷ Outside the overlap region no evidence for $K^*(890)$ is seen. The $(\Xi\pi\pi)$ vs $(\Xi\pi)_{I_Z=1/2}$ mass distribution in Fig. 1(b) shows two accumulations in the $\Xi(1530)$ band at $\Xi\pi\pi$ masses of 1830 and 1960 MeV. Outside the $\Xi(1530)$ band, no obvious structure is seen.

The $\Xi(1530)$ association of the two enhancements can be clearly seen in Fig. 2(a), where the cross-hatched histogram indicates events in the $\Xi(1530)$ band.⁸ In addition, the histogram shows the effects of removing the $\Xi(1530), K^*(890)$ overlap region; both peaks remain, although the $\Xi(1530)$ signal is reduced as expected.⁹ There exists no evidence for resonances in any other mass combination.¹⁰

It is reasonable, therefore, to hypothesize that the processes contributing to Reactions (1)-(4) are the following:

$$K^-p \rightarrow K\pi\pi\Xi, \quad (5)$$

$$\rightarrow K^*(890)\pi\Xi, \quad (6)$$

$$\rightarrow K\pi\Xi(1530), \quad (7)$$

$$\rightarrow K^*(890)\Xi(1530), \quad (8)$$

$$\rightarrow K\Xi(1820) \rightarrow K\pi\Xi(1530), \quad (9)$$

$$\rightarrow K\pi\pi\Xi, \quad (10)$$

$$\rightarrow K\Xi(1930) \rightarrow K\pi\Xi(1530), \quad (11)$$

$$\rightarrow K\pi\pi\Xi. \quad (12)$$

The data indicate [Figs. 1(a) and 1(b)] that processes (6), (10), and (12) are relatively small. Estimates for the relative fractions of each process (assuming no interference), and the masses and widths of resonances have been obtained using a maximum-likelihood method.¹¹

The procedure involved the following steps:

(a) Masses, widths, and fractions for the resonances $\Xi(1530)$, $K^*(890)$, $\Xi(1820)$, and $\Xi(1930)$ were first determined for all events from Reactions (1)-(4), irrespective of charge state.¹² Because of the presence of two ($I_Z = \frac{1}{2}$) $K\pi$ and $\Xi\pi$ mass combinations in channels (3) and (4), these events were included twice.

(b) Now events were split into two categories.

(i) Reactions (1) and (2) combined; (ii) Reactions (3) and (4) combined. Using the resonance parameters determined above, fractions for channels (5)-(12) were found for category (i) events.

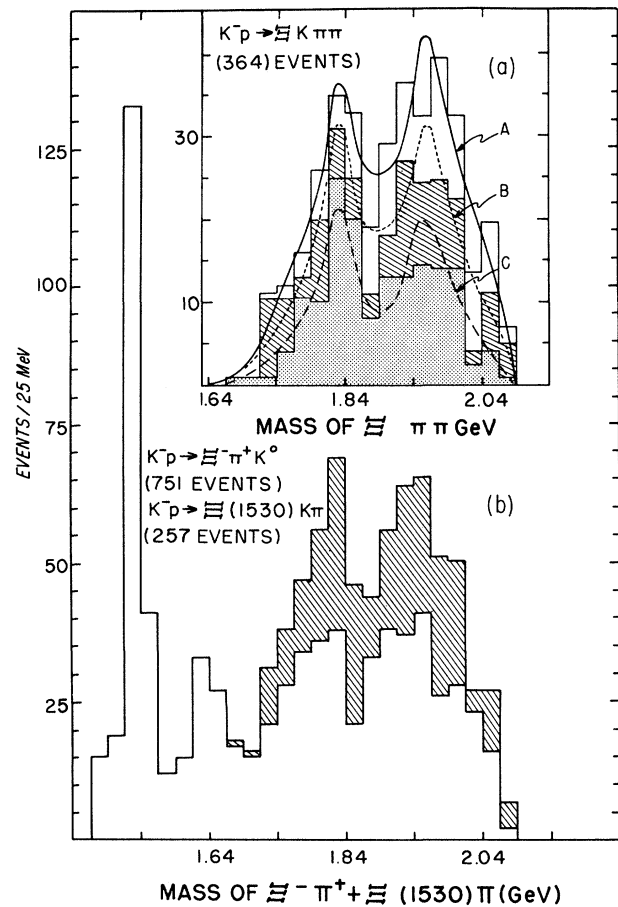


FIG. 2. (a) $M(\Xi\pi\pi)$ distribution and superimposed best-fit curves (see text) for (A) all events of reactions (1)-(4); (B) those events in the $\Xi(1530)$ region; (C) events in the $\Xi(1530)$ region but outside the $K^*(890)$ region (dotted). Events in both $K^*(890)$ and $\Xi(1530)$ are cross hatched. The curves result from the fit described in the text. (b) The combined spectra of $M(\Xi^-\pi^+)$ from $K^-p \rightarrow \Xi^-\pi^+K^0$ and $M(\Xi(1530)\pi)^0$ from $K^-p \rightarrow \Xi\pi\pi K$ (cross hatched).

The same was done for category (ii) except that independent fractions were introduced for the two charge states of $K^*(890)$ and $\Xi(1530)$ in channels (6), (7), (8), (9), and (11).

The percentages so obtained for the various charge states have finally been combined to obtain the overall percentage shown in Table I, which also gives the χ^2 's for various mass distributions.¹³ Phase space, $\Xi(1530)$, $K^*(890)$, and $\Xi(1530)\pi$ resonances dominate these reactions. The best-fit curves are superimposed on the mass projections of Fig. 1 and on Fig. 2(a). The resonance parameters obtained from these fits are discussed below.

(1) Masses and widths.—The determinations of mass and width in MeV for the $\Xi(1530)$ ($M = 1534.7$

Table I. Percentage contributions of each process to channels (1)-(4) resulting from maximum-likelihood fit (see text) and the χ^2 and number of bins for certain mass distributions.

Channel	Percentage ^a	Mass distribution	χ^2 /no. of bins ^b
(5) $K\pi\pi\Xi$	29.0 ± 3.3	$\Xi\pi(I_Z = \frac{1}{2})$	8.7/16
(6) $K^*(890)\pi\Xi$	8.0 ± 5.0	$K\pi(I_Z = \frac{1}{2})$	17.7/16
(7) $K\pi\Xi(1530)$	3.5 ± 6.1	$\Xi\pi\pi$	15.0/15
(8) $K^*(890)\Xi(1530)$	27.2 ± 3.8	$\Xi(1530)\pi$	15.2/14
(9) $K\Xi(1815) \rightarrow K\pi\Xi(1530)$	18.1 ± 3.5	$\Xi(1530)\pi^c$	13.9/13
(10) $K\Xi(1815) \rightarrow K\pi\pi\Xi$	1.3 ± 3.5	$\Xi\pi(I_Z = \frac{3}{2})$	13.8/15
(11) $K\Xi(1930) \rightarrow K\pi\Xi(1530)$	15.0 ± 3.5		
(12) $K\Xi(1930) \rightarrow K\pi\pi\Xi$	-2.1 ± 4.7		

^aPercentages refer to observed events with no corrections for visibility, other decay modes, or any losses.

^bThe χ^2 were obtained by averaging over 5 different binnings of the mass spectra subject to the requirement that at least 5 (theoretical) events should be found in each bin (≥ 25 MeV wide).

^c $K^*(890)$ removed.

± 1.1 , $\Gamma = 15 \pm 2$) and the $K^*(890)$ ($M = 891.8 \pm 2.2$, $\Gamma = 33 \pm 9$) are in satisfactory agreement with currently accepted values.¹⁴ Masses and widths for the $\Xi(1530)\pi$ resonances were determined to be $M(1820) = 1829 \pm 9$, $\Gamma(1820) = 52 \pm 34$, and $M(1930) = 1964 \pm 10$, $\Gamma(1930) = 60 \pm 39$ MeV. These values are in agreement with those determined from the $(\Xi\pi)^0$ mass spectrum in the $\Xi^-\pi^+K^0$ channel reported previously⁵ [$M(1820) = 1801 \pm 13$, $\Gamma(1820) = 78 \pm 33$ and $M(1930) = 1962 \pm 14$, $\Gamma(1930) = 147 \pm 55$ MeV]. The relevant mass spectra are shown in Figs. 2(a) and 2(b). As a result of this comparison it will be assumed that the $\Xi\pi$ and $\Xi(1530)\pi$ decay modes of the same particle are being observed.

(2) Isospin.—The channel $\Xi^-\pi^+K^0$ may be used to determine the isospin of the two $\Xi(1530)\pi$ resonant states. The method is independent of any experimental efficiency. The ratio $R = [\Xi^* \rightarrow \Xi^0(1530)\pi^0] / [\Xi^* \rightarrow \Xi^0(1530)\pi^0 + \Xi^-(1530)\pi^+]$, where $\Xi^* \rightarrow \Xi^0(1530) \rightarrow \Xi^-\pi^0K^+$, is predicted to be 0.5 for $I = \frac{1}{2}$ and 0.8 for $I = \frac{3}{2}$. Using binomial statistics for 28 $\Xi(1820)$ and 23 $\Xi(1930)$ events the uncertainty in R will be approximately ± 0.1 in both cases. From our maximum-likelihood fit we obtain $R = 0.2$ for the $\Xi(1820)$ and $R = 0.6$ for the $\Xi(1930)$. These results favor an $I = \frac{1}{2}$ assignment for both states.

(3) Branching ratios.—A comparison of our results for the $\Xi^-\pi^+K^0$ channel [see Ref. 5 and Fig. 2(b)] with those from our analysis of channels (1)-(4) yields the following branching fractions (assuming $I = \frac{1}{2}$)¹⁵:

$$\begin{aligned} \Xi(1820) &\rightarrow \Xi\pi: \Xi(1530)\pi: \Xi\pi\pi \\ &= 1.5^{+0.6}_{-0.4}: 1.0: <0.3 \pm 0.5, \end{aligned}$$

$$\Xi(1930) \rightarrow \Xi\pi: \Xi(1530)\pi: \Xi\pi\pi$$

$$= 2.8^{+0.7}_{-0.6}: 1.0: <0.0 \pm 0.3.$$

These numbers are fully corrected for visibility, geometrical biases, scanning efficiency,¹⁶ and other charge states.

(4) Conclusions.—We have demonstrated that $\Xi(1820)$ and $\Xi(1930)$ decay into $\Xi(1530)\pi$ as well as $\Xi\pi$ channels. The result of adding the $(\Xi^-\pi^+)$ and $(\Xi(1530)\pi)^0$ spectra is shown in Fig. 2(b).¹⁷ The combined spectrum firmly establishes the $\Xi(1820)$ and $\Xi(1930)$ in our data. The weighted averages for the mass and width of these resonances in MeV, using the data from this experiment only, are $(1820 \pm 7, 64 \pm 23)$ and $(1963 \pm 8, 89 \pm 33)$. It should be noted that the number of events in each resonance, in the combined spectrum, is ~ 100 events for $\Xi(1820)$ and ~ 200 events for $\Xi(1930)$. The upper resonance has a mass 30 MeV higher than in the present world summary.¹⁴

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⁶Events in Reaction (4) include those that have failed to fit Reaction (3) for technical reasons, although the Λ decay products are visible. After scanning efficiencies and geometrical corrections are taken into account, the ratio of visible to invisible Λ events in Reactions (3) and (4) is consistent with accepted values of the Λ branching ratio [see N. Barash-Schmidt et al., *Rev. Mod. Phys.* **41**, 109 (1969)]. Our conclusions regarding the properties of resonances in these channels are not significantly altered by the inclusion or exclusion of ambiguous events.

⁷For channels (3) and (4), two $I_z = \frac{1}{2}$ $K\pi$ and $\Xi\pi$ combinations are possible, and both have been included in Figs. 1 (a) and 1 (b).

⁸The $\Xi(1530)$ band is defined as $1.500 \text{ GeV} < M(\Xi\pi) < 1.570 \text{ GeV}$; the $K^*(890)$ band is defined as $0.86 \text{ GeV} < M(K\pi) < 0.92 \text{ GeV}$.

⁹For kinematic reasons, events with $M(\Xi\pi\pi) \approx 1.960$

GeV have a high probability of appearing in the $\Xi(1530)$, $K^*(890)$ overlap region. Thus, removal of overlap events unavoidably diminishes the enhancement at $M(\Xi\pi\pi) = 1.960$.

¹⁰The total center-of-mass energy of 2.56 GeV does not allow the production of the ρ in these reactions. The two $I = \frac{3}{2}$ combinations, $\Xi^0\pi^+$ and $\Xi^-\pi^-$, show no significant structure.

¹¹The fitting programs MASTERFEIT [see B. T. Meadows, Syracuse Technical Report No. T-6901, 1969 (unpublished)] and MURLEBERT [see J. Friedman, Alvarez Group Programming Note, P-156, 1966 (unpublished)] were used. No significant differences were noted. For full details of the procedure adopted see R. J. Hemingway and B. T. Meadows, Intramural Report No. I11-69, 1969 (unpublished). Noninterfering s -wave Breit-Wigner resonances were always assumed.

¹²In both $K^*(890)$ and $\Xi(1530)$ two charge states are present. For the $\Xi(1530)$ the observed neutral to negative ratio is approximately $\frac{4}{3}$.

¹³We note that in the $M(\Xi\pi\pi)$ and $M(\Xi(1530)\pi)$ distributions, $\frac{1}{3}$ of the contribution to the χ^2 results from a 10-12 event excess in the region of 2030 MeV where a resonance has been reported, e.g., see Ref. 3.

¹⁴Barash-Schmidt et al., Ref. 6. The experimental resolution was not unfolded from the width.

¹⁵The uncharged Ξ states were used to determine these ratios. Three-body decay into $\Xi^0\pi^0\pi^0$ was estimated as generously as possible on the assumption that $I = \frac{1}{2}$ for both Ξ 's.

¹⁶See R. Hemingway and H. Whiteside, Maryland Technical Report No. 70-065, 1969 (unpublished) for details regarding these corrections.

¹⁷The structure at 1630 MeV in Fig. 2b has been discussed in Ref. 5.

STUDY OF THRESHOLD ENHANCEMENTS IN $\rho^0\pi^-$ AND $f^0\pi^-$ SYSTEMS FROM THE REACTION $\pi^-p \rightarrow p\pi^+\pi^-\pi^-$ AT 6 GeV/c *

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We have observed, in a high-statistics experiment, well-defined threshold enhancements in the $\rho^0\pi^-$ and $f^0\pi^-$ systems at 1120 and 1645 MeV, respectively. We have searched for other dominant decay modes of these enhancements and found none. The question of whether these objects are threshold enhancements of kinematic origin or resonances is examined.

Broad boson enhancements have been observed at threshold in high-energy quasi-two-body final states recoiling against targetlike nucleons in various reactions: A_1^+ in the $\rho\pi$ system and A_3^+ in the $f\pi$ system from π^+ reactions¹; " Q^+ " in the $K^*(890)\pi$ system and L^+ in the $K^*(1420)\pi$ system from K^+ reactions.² The question of whether these enhancements are real resonances or effects of kinematic origin has been examined from experimental as well as theoretical points of

view.³ In particular, recent developments in the concept of duality in relating the resonant behavior of the imaginary part of the s -channel amplitudes to the t -channel Regge-pole exchanges⁴ further support the resonance interpretation of these low-mass enhancements.

In this Letter we present an analysis of the $\rho^0\pi^-$ and $f^0\pi^-$ systems near their threshold regions from the reaction $\pi^-p \rightarrow p\pi^+\pi^-\pi^-$ at 6 GeV/c with better statistics than any single reported ex-