WHY SABINE HOSSENFELDER IS WRONG: CASE ELECTROMAGNETISM AS A PURELY GEOMETRIC THEORY

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1. Introduction

Ever since the discovery of general relativity, it remained an open challenge to determine whether its scope can be extended to also include electromagnetic forces and interactions. Such an extension has been Einstein's dream of unifying gravity with electromagnetism. We believe that our article titled "Electromagnetism as a purely geometric theory" [1] represents a major progress in this hotly debated field. The core proposition of reference [1] is that the spacetime metric tensor, which defines spacetime lengths and curvature, encodes electromagnetic fields and charges starting from the following simple relationship:

$$g_{\mu\nu} = \eta_{\mu\nu} + A_\mu A_\nu \tag{1.1}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, and A is the electromagnetic four-potential. From this starting point, Maxwell's equation is derived into a particularly simple geometric form:

$$\Delta A_\nu = J_\nu. \tag{1.2}$$

Equations 1.1 and 1.2 certainly pass the test of Occam's razor. The publication of reference [1] therefore received the appropriate attention, and what remains is to validate the proposed theory. In this context, a controversy has been raised by social media influencer Sabine Hossenfelder, who alleged that our derivations contain several mathematical errors and engaged in a theatrical defamation of us and the publisher [3]. Normally, social media influencers are not part of the scientific discourse. In this case, a debunking of Sabine Hossenfelder's allegations is warranted because her presentation has been viewed by 320 000 science enthusiasts, many of whom are physicists. It appears from the comments section of [3] that most viewers, including physicists, assume or believe that Sabine Hossenfelder's allegations are true. For this reasons, we clarify each allegation in the following sections.

2. Factorization is not dividing by a vector

We factorize equation (39) of reference [1] in order to arrive at equation (40). Specifically, by linearity of the scalar product, the factorized equation (39) takes the following form:

$$A^\nu(-J_\nu + \Delta A_\nu) = 0 \tag{2.1}$$

The above equation is fulfilled when either the expanded scalar product is zero, $A^\nu = 0$ or $(-J_\nu + \Delta A_\nu) = 0$. Selecting one of these solutions does not involve any dividing by a vector, contrary to what is being alleged in [3]. In other words, a factorization of an equation does not mean that we are dividing by anything.

One physically meaningful class of solutions is obviously given by the second factor of the above equation:

$$-J_\nu + \Delta A_\nu = 0 \tag{2.2}$$

which is equation (40) of our article.

The other $A^\nu = 0$ solution is not of physical interest, because that gives just the empty, flat Minkowski space. Furthermore, it is odd to accuse that we are not looking at all the solutions, because the whole point of the section is to provide one special class of solutions, which is the Maxwell source equations.

3. Tensor trace calculations

It is alleged in [3] that we are making a mathematical mistake whenever we set two dummy summation indices to the same value. While it is true that in general it is dangerous, we have equated two summation indices only where it is appropriate to do so. In the following, we explain each example where we can set two summation indices to the same value. By taking her time to go through the following analyses, or by asking us for clarifications, the author of [3] might have realized that her allegations are baseless.

3.1. **The derivation of equation (25) from equation (22).** We start from equation (22) of [1], which is the geodesic equation:

$$\frac{d^2x^\mu}{d\tau^2} + \left(g^{\mu\alpha}\frac{1}{2}\left(A_\nu F_{\alpha\lambda} + A_\lambda F_{\alpha\nu}\right)\right)\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau} = 0 \tag{3.1}$$

Equation (24) of our article is $\rho = -A_\nu \frac{dx^\nu}{d\tau}$, which we substitute into the above equation, and obtain:

$$\frac{d^2x^\mu}{d\tau^2} + \left(g^{\mu\alpha}\frac{1}{2}\left(-\rho F_{\alpha\lambda}\frac{dx^\lambda}{d\tau} - \rho F_{\alpha\nu}\frac{dx^\nu}{d\tau}\right)\right) = 0 \tag{3.2}$$

As now λ and ν are dummy indices, it is clear that the two covectors inside the brackets are the same, so that we may simplify the above equation into the following form:

$$\frac{d^2x^\mu}{d\tau^2} + \left(g^{\mu\alpha}\left(-\rho F_{\alpha\nu}\frac{dx^\nu}{d\tau}\right)\right) = 0 \tag{3.3}$$

Recognizing that the current density is given by $J^\nu = \rho\frac{dx^\nu}{d\tau}$, the above equation is the Lorentz force law equation (25) of [1]. Therefore the original approach in our paper is appropriate.

3.2. **The derivation of equation (29) from equation (28).** We can in principle demand that the eigenvalue equation characterizing a Weyl connection holds for the diagonal elements of the metric tensor, which justifies retaining only the $\nu = \lambda$ components of equation (28). We thus obtain equation (29) of [1].

3.3. **The derivation of equation (37) from equation (31).** We can justify our result (That is, obtaining Maxwell's source equation) by first choosing $\mu = \nu$ so no already contracted indices.

Then we obtain from equation (31)

$$\nabla_\sigma A_\nu \nabla^\sigma A_\nu + A_\nu \Delta A_\nu + \nabla_\sigma A_\nu \nabla^\sigma A_\nu + A_\nu \Delta A_\nu = 0. \tag{3.4}$$

Next, raise an index by multiplying with $g^{\sigma\nu}$ (one can always add a constant metric, the Minkowski metric to the singular part to make the metric invertible, but this is technics)

$$\nabla^\nu A_\nu \nabla^\sigma A_\nu + A^\sigma \Delta A_\nu + \nabla^\nu A_\nu \nabla^\sigma A_\nu + A^\sigma \Delta A_\nu = 0. \tag{3.5}$$

Now we can see that the contractions and dummy indices disappear, instead of those, we have just the scalar charge density fields:

$$-\rho\nabla^\sigma A_\nu + A^\sigma \Delta A_\nu - \rho\nabla^\sigma A_\nu + A^\sigma \Delta A_\nu = 0. \tag{3.6}$$

And finally, contract with setting $\sigma = \nu$, to obtain:

$$\rho^2 + A^\nu \Delta A_\nu + \rho^2 + A^\nu \Delta A_\nu = 0. \tag{3.7}$$

We have thus obtained equation (37) in the published paper.

4. Light-speed elementary charge circulation

The light-speed motion of elementary electric charges was first proposed by Gregory Breit (1928), and secondly by Erwin Schrödinger (1930). In brief, both scientists propose that elementary charges have a light-speed oscillatory motion around their mean location, and Schrödinger introduced the Zitterbewegung expression in reference to this light-speed oscillatory motion. The amplitude of this oscillation is the electron's reduced Compton radius, and it is thought to be the origin of electron spin [4]. Because this oscillation radius is very small, from our macroscopic perspective we do not directly perceive the electron's light-speed oscillation; the directly perceived electron speed is the much slower displacement of its mean location. In his Nobel lecture, held in 1933, Paul Dirac also explained that the electron comprises an elementary charge that moves at the speed of light: "It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light." Following in the footsteps of these well-respected scientists, between 1933 and the present day numerous physicists have been studying the properties and dynamics associated with the electron's light-speed oscillatory motion. A comprehensive overview of the most influential publications on this topic can be found in reference [5].

Sabine Hossenfelder is of course entitled to deny any oscillation of elementary charges. However, it is misleading for her to project a personal opinion as scientific consensus. When she makes a declarative statement on elementary charges not moving at the speed of light [3], she should add context such as "this is my personal opinion, and I am aware that it is opposed by the opinions of Breit, Schrödinger, Dirac, Hestenes, and many others".

5. The use of vector potential in foundational equations

It is also alleged by Sabine Hossenfelder that we make a mistake in equation (24) of our article, because in her opinion a physically measurable quantity should not depend on the electromagnetic vector potential [3]. That is an ad-hoc claim, which is contradicted by various experimentally validated physics equations. For example, the Aharonov-Bohm effect is described by the electric and magnetic Aharonov-Bohm equations, which directly carry the A_ν field. The well-known Dirac equation also directly carries the eA_ν field term in its Hamiltonian operator. It is therefore surprising that the 320 000 viewers of [3] did not object to Hossenfelder's denial of the electromagnetic vector potential.

References

[1] J. Lindgren et al. "Electromagnetism as a purely geometric theory", Journal of Physics: Conference Series (2025)
[2] Vassallo G. et al. Maxwell's equations and Occam's razor, Journal of Condensed Matter Nuclear Science, 25.1 (2017)
[3] https://www.youtube.com/watch?v=ll2hrh_BJfQ
[4] D. Hestenes "Zitterbewegung structure in electrons and photons" (2020)
[5] https://www.zitter-institute.org/p/zitter-repository.html
[6] S. H. Simon "Lecture Notes for Quantum Matter", Oxford University (2025)