large number of such repetitions into our teaching. It may bore a few poor students, but almost all benefit.

IX. THE IMPORTANCE OF CALCULATING WITH NUMBERS

The world has changed quite a bit in the past 30 or 40 years. When I was an undergraduate we learned that there are only four angles in this world, namely, 30°, 45°, 60°, and 90°. Furthermore, all measurements are divisible by 2, often by 3 and 4, and, curiously, not infrequently by 49. It came as something of a surprise, when I embarked on experimental research, to find that most measurements are embarrassingly inelegant numbers, and that angles, as often as not, wander somewhere between those canonical values we learned in class.

I understand why my student problems had such remarkably simple numbers. It was just that nobody liked long division, and the alternatives were few.

Of course, we did have pocket calculators, or, more accurately, hip calculators. But they were hard to use, required a fair amount of manual dexterity to get results accurate to three figures. They were slow, and very expensive. My present shirt pocket calculator, whose batteries have already lasted two years, not only gives me nine figures and hyperbolic functions, but even does arithmetic in hexadecimal. It cost $14.29. When students grumble about the expense, I delight to tell them that my 1945 log log duplex trig calculator, required on every test, cost me $176 (in 1985 dollars, using an average inflation rate of 5% per annum).

My point is this. Calculating power today is dirt cheap. It costs far less than textbooks and it lasts from one course to another. It gives us the opportunity to teach the physics of the real world rather than the physics of the textbook. Our students, furthermore, at least our technically inclined students, will spend their lives making use of these calculators.

This needs to be recognized in what we do in our calculus-based physics. Thirty, 60, and 90 ought to be reduced to their proper place. In my classes, tests, if not textbook problems, have angles like 27.6°. Automobiles have speeds of 37 km/h. Electrons move in orbits of radius 0.26 centimeters. The only difficulties students have with this is that too frequently their calculations seem to be accurate to one part in ten to the ninth.

All this is fine for the science and engineering students. What about the liberal arts students? Years ago, I would not have dreamed of asking them to buy slide rules. I hesitate now to ask them to have calculators, yet I note that almost all do. I continue to give them problems with nice numbers, yet I find them using a calculator to divide 8 by 4. I'm beginning to think that they too should always deal with real-world numbers. If they have to use a calculator to divide 8 by 4, they might as well be dividing 8.63 by 4.79.

By now I have run the device of numbers into the ground. It has given me a handy framework to air my grievances about and my hopes for physics teaching. I hope I will hear more about these dirty problems of physics teaching in less than ideal circumstance from the rest of you. Let me thank the AAPT once again for giving me this award. Thank you all for hearing me out.

What is spin?

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According to the prevailing belief, the spin of the electron or of some other particle is a mysterious internal angular momentum for which no concrete physical picture is available, and for which there is no classical analog. However, on the basis of an old calculation by Belinfante [Physica **6**, 887 (1939)], it can be shown that the spin may be regarded as an angular momentum generated by a circulating flow of energy in the wave field of the electron. Likewise, the magnetic moment may be regarded as generated by a circulating flow of charge in the wave field. This provides an intuitively appealing picture and establishes that neither the spin nor the magnetic moment are "internal"—they are not associated with the internal structure of the electron, but rather with the structure of its wave field. Furthermore, a comparison between calculations of angular momentum in the Dirac and electromagnetic fields shows that the spin of the electron is entirely analogous to the angular momentum carried by a classical circularly polarized wave.

I. INTRODUCTION

When Goudsmit and Uhlenbeck proposed the hypothesis of the spin of the electron, they had in mind a mechanical picture of the electron as a small rigid body rotating about its axis. Such a picture had earlier been considered by Kronig and discarded on the advice of Pauli, Kramers, and Heisenberg, who deemed it a fatal flaw of this picture that the speed of rotation—calculated from the magnitude of the spin and a plausible estimate of the radius of the electron—was in excess of the speed of light. However, the great success of the spin hypothesis in explaining the Zeeman effect and the doublet structure of spectral lines quickly led to its acceptance.[1] Since the naive mechanical picture of spin proved untenable, physicists were left with the concept of spin minus its physical basis, like the grin of the Cheshire cat. Pauli pontificated that spin is "an essentially quantum-mechanical property,...a classically not describable two-valuedness"[2] and he insisted that the lack of a concrete picture was a satisfactory state of affairs:

> After a brief period of spiritual and human confusion, caused by a provisional restriction to 'Anschaulichkeit', a general agreement was reached following the substitution of abstract mathematical symbols, as for instance psi, for concrete pictures. Especially the concrete picture of rotation has been replaced by mathematical characteristics of the representations of rotations in three-dimensional space.[3]

Thus physicists gradually came to regard the spin as an abstruse quantum property of the electron, a property not amenable to physical explanation.

Judging from statements found in modern textbooks on atomic physics and quantum theory, one would think our understanding of spin (or the lack thereof) has not made any progress since the early years of quantum mechanics. The spin is usually said to be a nonorbital, "internal," "intrinsic," or "inherent" angular momentum (the words are often used interchangeably, although they should not be), and it is often treated as an irreducible entity that cannot be explained further. Sometimes the (unsubstantiated) suggestion is made that the spin is due to an (unspecified) internal structure of the electron.[4] And sometimes the consolation is offered that the spin arises in a natural way from Dirac's equation[5] or from the analysis of the representations of the Lorentz group. It is true that the Dirac equation contains a wealth of information about spin: The equation tells us that the spinor wavefunctions are indeed endowed with a spin angular momentum of $\hbar/2$, it supplies the mathematical description of the kinematics of a free-electron or other particle of spin one-half, and—in conjunction with the principle of minimal coupling—it supplies the equations governing the dynamics of a charged particle immersed in a electromagnetic field, equations which directly yield the correct value of the gyromagnetic ratio for the electron. It is also true that the analysis of the representations of the Lorentz group is very informative: The analysis tells us that the quantum-mechanical wavefunctions must be certain types of tensors or spinors characterized by a value of the mass and (if the mass is not negative) an integer or half-integer value of the spin. But in all of this the spin merely plays the role of an extra, nonorbital angular momentum of unknown etiology. Thus the mathematical formalism of the Dirac equation and of group theory demands the existence of the spin to achieve the conservation of angular momentum and to construct the generators of the rotation group, but fails to give us any understanding of the physical mechanism that produces the spin.

The lack of a concrete picture of the spin leaves a grievous gap in our understanding of quantum mechanics. The prevailing acquiescence to this unsatisfactory situation becomes all the more puzzling when one realizes that the means for filling this gap have been at hand since 1939, when Belinfante[6] established that the spin could be regarded as due to a circulating flow of energy, or a momentum density, in the electron wave field. He established that this picture of the spin is valid not only for electrons, but also for photons, vector mesons, and gravitons—in all cases the spin angular momentum is due to a circulating energy flow in the fields. Thus contrary to the common prejudice, the spin of the electron has a close classical analog: It is an angular momentum of exactly the same kind as carried by the fields of a circularly polarized electromagnetic wave. Furthermore, according to a result established by Gordon[7] in 1928, the magnetic moment of the electron is due to the circulating flow of charge in the electron wave field. This means that neither the spin nor the magnetic moment are internal properties of the electron—they have nothing to do with the internal structure of the electron, but only with the structure of its wave field.

Unfortunately, this clear picture of the physical origin of the spin and of the magnetic moment has not received the wide recognition it deserves, perhaps because neither Belinfante nor Gordon loudly proclaimed that their calculations provided a new physical explanation of the spin and of the magnetic moment. These calculations are sometimes reproduced in texts on quantum field theory,[8] but usually without any commentary on their physical interpretation. In the present paper, it is my objective to revive these forgotten explanations of the spin and the magnetic moment in the hope that the intuitive picture of circulating energy and charge will become part of the lore learned by all students of physics. I want to emphasize that, in contrast to some other attempts at explaining the spin,[9] the present explanation is completely consistent with the standard interpretation of quantum mechanics.

A crucial ingredient in Belinfante's calculation of the spin angular momentum is the use of the symmetrized energy-momentum tensor. It is well known that in a field theory we can construct several energy-momentum tensors, all of which satisfy the conservation law $\partial_\nu T^{\mu\nu} = 0$, and all of which yield the same net energy ($\int T^{00}\, d^3x$) and momentum ($\int T^{k0}\, d^3x$) as the canonical energy-momentum tensor.[10] These diverse energy-momentum tensors differ by terms of the form $\partial_\alpha U^{\mu\nu\alpha}$, which are antisymmetric in the last two indices ($U^{\mu\nu\alpha} = -U^{\mu\alpha\nu}$), and therefore identically satisfy the conservation law $\partial_\nu \partial_\alpha U^{\mu\nu\alpha} = 0$. Belinfante showed that by a suitable choice of the term $\partial_\alpha U^{\mu\nu\alpha}$, it is always possible to construct a *symmetrized* energy-momentum tensor ($T^{\mu\nu} = T^{\nu\mu}$). The symmetrized energy-momentum tensor has the distinctive advantage that the angular momentum calculated directly from the momentum density T^{k0} is a conserved quantity (in the absence of external torques). This means that the momentum density gives rise to both orbital angular momentum and spin angular momentum. If instead of the symmetrized energy-momentum tensor, we were to use the unsymmetrized canonical energy-momentum tensor, then the momentum density would not give rise to the spin angular momentum. This does not mean that the spin would vanish from the theory—an examination of the conservation law for angular momentum shows that the spin emerges as a mysterious extra quantity that must be added to the orbital angular momentum to achieve conservation—but the simple and clear physical mechanism underlying spin would vanish. I will take it for granted that the symmetrized energy-momentum tensor is the correct energy-momentum tensor. As emphasized by Rosenfeld,[11] this is not an arbitrary choice; rather, it is compellingly demanded by Einstein's theory of gravitation, which is compatible only with a symmetric energy-momentum tensor.

II. SPIN IN THE ELECTROMAGNETIC FIELD

The energy flow in the electromagnetic field (in vacuum) is given by the Poynting vector $\mathbf{E}\times\mathbf{B}/\mu_0$. The momentum density $\mathbf{G}$ is the same, except for a factor of $1/c^2$:

$$\mathbf{G} = \mathbf{E}\times\mathbf{B}/\mu_0 c^2. \tag{1}$$

In an infinite plane wave, the **E** and **B** fields are everywhere perpendicular to the wave vector and the energy flow is everywhere parallel to the wave vector. However, in a wave of finite transverse extent, the **E** and **B** fields have a component parallel to the wave vector (the field lines are closed loops) and the energy flow has components perpendicular to the wave vector. For instance, Fig. 1 shows the time-average transverse energy flow in a circularly polarized wave propagating in the z direction; the wave has a finite extent in the x and y directions and it has cylindrical symmetry about the z axis. Besides the circulating energy flow shown in Fig. 1, the wave has a translational flow in the z direction; hence the net energy flow is helical. The circulating energy flow in the wave implies the existence of angular momentum, whose direction is along the direction of propagation. This angular momentum is the spin of the wave. If the wave is not centered on the origin or if the wave is asymmetric, then the translational energy flow implies the existence of an additional "orbital" angular momentum.

To obtain an expression for the net angular momentum in an arbitrary wave packet, we begin by expressing the momentum density as a sum of two terms:

$$\begin{aligned}\mathbf{E}\times\mathbf{B}/\mu_0 c^2 &= \mathbf{E}\times(\nabla\times\mathbf{A})/\mu_0 c^2\\ &= [E^n \nabla A^n - (\mathbf{E}\cdot\nabla)\mathbf{A}]/\mu_0 c^2.\end{aligned} \tag{2}$$

Correspondingly, the net angular momentum is a sum of two terms:

$$\begin{aligned}\mathbf{J} = &\frac{1}{\mu_0 c^2}\int \mathbf{x}\times(E^n \nabla A^n)\,d^3x\\ &+\frac{1}{\mu_0 c^2}\int \mathbf{x}\times[-(\mathbf{E}\cdot\nabla)\mathbf{A}]\,d^3x.\end{aligned} \tag{3}$$

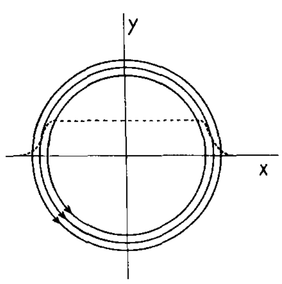

Fig. 1. This pattern of circular flow lines represents the time-average energy flow, or the momentum density, in a circularly polarized electromagnetic wave packet. On a given wave front, say $z = 0$, the fields are assumed to be constant within a circular area and to decrease to zero outside of this area (the dashed line gives the field amplitude as a function of radius). The energy flow has been calculated from an approximate solution of Maxwell's equations. The picture only shows the flow in the transverse directions. The flow in the longitudinal direction is much larger; the net flow is helical.

With an integration by parts and with $\nabla\cdot\mathbf{E} = 0$ this becomes

$$\mathbf{J} = \frac{1}{\mu_0 c^2}\int \mathbf{x}\times(E^n \nabla A^n)\,d^3x + \frac{1}{\mu_0 c^2}\int \mathbf{E}\times\mathbf{A}\,d^3x. \tag{4}$$

One big advantage of Eq. (4) over Eq. (2) is that the former permits a direct and simple evaluation of the spin in a quasiplane wave consisting of an inner region of large volume within which the amplitudes of the electric and magnetic fields are nearly uniform, surrounded by an outer region of small volume within which the amplitudes are decreasing and nonuniform. According to Eq. (3), the evaluation of the spin angular momentum in such a wave requires a knowledge of the (very complicated) fields in the outer region. But according to Eq. (4), the spin receives most of its contribution from the inner region, and the outer region can be neglected, vastly simplifying calculation. The mathematical equivalence between these two ways of calculating the spin is reminiscent of the equivalence between the two ways of calculating the magnetic field of a uniformly magnetized body: either by integrating the magnetization over the volume of the body, or else by integrating the Amperian magnetization current over the surface of the body.

The first term in Eq. (4) represents the orbital angular momentum, and the second term the spin. To justify this interpretation, consider a circularly polarized plane wave with a vector potential

$$\mathbf{A} = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})(iE_0/\omega)e^{i\omega t - i\omega z/c}. \tag{5}$$

Of course, at the edge of the wave this vector potential will have to be modified to fit the decreasing, nonuniform fields; but for a quasiplane wave we can neglect this modification. The time-average values of the integrals in Eq. (4) are then

$$\mathbf{L} = \frac{1}{2\mu_0 c^2}\int \mathrm{Re}(\mathbf{x}\times E^n \nabla \mathbf{A}^{*n})\,d^3x = \frac{1}{\mu_0 c^3}\,\mathbf{x}\times(\hat{\mathbf{z}}E_0^2)\,d^3x \tag{6}$$

and

$$\mathbf{S} = \frac{1}{2\mu_0 c^2}\int \mathrm{Re}(\mathbf{E}\times\mathbf{A}^*)\,d^3x = \pm\frac{1}{\mu_0 c^2}\int \frac{\hat{\mathbf{z}}E_0^2}{\omega d^3x}. \tag{7}$$

The first of these expressions is independent of polarization, and it is exactly what we expect for the orbital angular momentum of the plane wave. The second expression is independent on the polarization, and we must therefore identify it as the spin. Note that the individual integrals in Eq. (4) are not gauge invariant. This means that the separation into orbital and spin parts provided by Eq. (4) is tied to a particular choice of gauge, viz., the choice of gauge specified by Eq. (5). Of course, the net expression (3) for the angular momentum is gauge invariant, and if we want to calculate in some arbitrary gauge, then we must return to this net expression.

According to Eq. (4), the mechanism underlying spin is not essentially different from that underlying orbital angular momentum: Both forms of angular momentum arise from the momentum density in the fields. The distinction between spin and orbital contributions to the total angular momentum merely results from the independence of these contributions: The portion of the momentum density that gives rise to the spin can be reversed independently of the portion that gives rise to the orbital angular momentum.

The energy in the wave (5) is

$$U=\frac{1}{2\mu_0c^2}\int \mathrm{Re}(\mathbf{E}\cdot\mathbf{E}^*)d^3x=\frac{1}{\mu_0c^2}\int E_0^2\,d^3x. \quad (8)$$

Hence the ration of spin to energy is

$$S_z/U=1/\omega. \quad (9)$$

If we normalize the wave so that its energy is one quantum ($U=\hbar\omega$), then its spin will be $S_z=\hbar$. The magnitude of the spin or the photon is therefore uniquely determined by Maxwell's equations and the condition of energy quantization. However, in order to obtain the complete mathematical formalism for the quantum-mechanical description of the spin (commutation relations), we need to quantize the electromagnetic field. This requires the methods of quantum field theory.

III. SPIN IN THE DIRAC FIELD

According to the symmetrized energy-momentum tensor, the momentum density in the Dirac field is[12]

$$\mathbf{G}=\frac{\hbar}{4i}\left(\psi^\dagger\nabla\psi-\frac{1}{c}\,\psi^\dagger\boldsymbol{\alpha}\,\frac{\partial\psi}{\partial t}\right)+\mathrm{hc}, \quad (10)$$

where hc stands for the hermitian conjugate of the preceding term. The time derivative appearing in Eq. (10) can be eliminated by means of the Dirac equation

$$\frac{1}{c}\,\frac{\partial\psi}{\partial t}=\left(-\boldsymbol{\alpha}\cdot\nabla+\frac{mc^2}{i\hbar}\,\beta\right)\psi, \quad (11)$$

which gives

$$\mathbf{G}=(\hbar/4i)\,[\psi^\dagger\nabla\psi+\psi^\dagger\boldsymbol{\alpha}(\boldsymbol{\alpha}\cdot\nabla)\psi]+\mathrm{hc}. \quad (12)$$

The commutation relations for the matrices α_k then lead to

$$\mathbf{G}=(\hbar/2i)\ [\psi^\dagger\nabla\psi-(\nabla\psi^\dagger)\psi]+(\hbar/4)\nabla\times(\psi^\dagger\boldsymbol{\sigma}\psi), \quad (13)$$

where $\sigma_1=-i\alpha_2\alpha_3$, $\sigma_2=-i\alpha_3\alpha_1$, and $\sigma_3=-i\alpha_1\alpha_2$.

The first term in this momentum density is associated with the translational motion of the electron, whereas the second term is associated with circulating flow of energy in the rest frame of the electron. For instance, consider the Gaussian packet

$$\psi=(\pi d^2)^{-3/4}e^{-(1/2)r^2/d^2}w^1(0) \quad (14)$$

which represents, in the nonrelativistic limit ($d\gg\hbar/mc$), an electron of spin up with zero expectation value of the momentum. Then the first term in Eq. (13) is zero, and the second term is

$$\mathbf{G}=\frac{\hbar}{4}\left(\frac{1}{\pi d^2}\right)^{3/2}\frac{e^{-r^2/d^2}}{d^2}\,(-2y\,\hat{\mathbf{x}}+2x\hat{\mathbf{y}}). \quad (15)$$

Figure 2 shows the flow lines for the energy. As in the case of the electromagnetic wave, such a circulating flow of energy will give rise to an angular momentum. This angular momentum is the spin of the electron.

For an arbitrary wave packet, the net angular momentum is

$$\mathbf{J}=\int\frac{\hbar}{2i}\,\mathbf{x}\times[\psi^\dagger\nabla\psi-(\nabla\psi^\dagger)\psi]d^3x$$
$$+\int\frac{\hbar}{4}\mathbf{x}\times[\nabla\times(\psi^\dagger\boldsymbol{\sigma})\psi]d^3x. \quad (16)$$

Here it is convenient to expand the triple cross product in the second integral into two dot products and then integrate both of these by parts. This gives the result

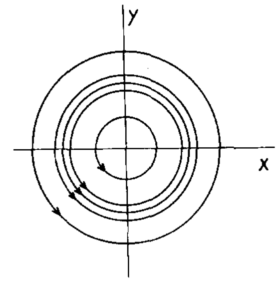

Fig. 2. This pattern of flow lines represents the energy flow in the spinor wave packet (14) describing an electron with spin in the z direction and with zero average momentum.

$$\mathbf{J}=\frac{\hbar}{2i}\int\mathbf{x}\times[\psi^\dagger\nabla\psi-(\nabla\psi^\dagger)\psi]d^3x$$
$$+\frac{\hbar}{2}\int\psi^\dagger\boldsymbol{\sigma}\psi\,d^3x. \quad (17)$$

The first term is the orbital angular momentum and the second term is the spin:

$$\mathbf{S}=\frac{\hbar}{2}\int\psi^\dagger\boldsymbol{\sigma}\psi\,d^3x. \quad (18)$$

As in the case of the electromagnetic wave, we can justify this identification by noting that the first term is independent of the spinorial state and has the form expected of the orbital angular momentum, whereas the second term is dependent on the spinorial state. Note that, since the spin defined by Eq. (18) is the expectation value of the quantum-mechanical operator $\boldsymbol{\sigma}$, the operator representing the spin must be

$$\mathbf{S}_{\mathrm{op}}=(\hbar/2)\boldsymbol{\sigma}. \quad (19)$$

Thus the spin operator obeys all the usual commutation relations, and we can deduce all the familiar quantum-theoretical properties of the spin from our equations. In particular, the eigenvalues of any component of the spin, say the z component, are $\pm\hbar/2$ and substitution of the corresponding (normalized) eigenfunction into Eq. (18) yields the value $\pm\hbar/2$ for the integral representing S_z. What is important here is not so much the numerical value of this result—there are a variety of ways of establishing that the Dirac spinors correspond to spin $\hbar/2$—but rather the underlying physical picture of the spin as due to a circulating energy flow in the Dirac field.

IV. THE MAGNETIC MOMENT OF THE ELECTRON

In Sec. III we saw that the spin can be attributed to a circulating flow of energy in the wave field. It will therefore come as no surprise that the magnetic moment of the electron similarly can be attributed to a circulating flow of electric charge in the wave field, a circulating flow of charge that exists even for an electron at rest. To recognize the existence of this flow, we separate the standard electric current density $-e\bar{\psi}\gamma^k\psi$ of the Dirac field into two parts by means of the well-known Gordon decomposition formula, which is a consequence of the Dirac equation for a free-electron:[7,13]

$$-ec\bar{\psi}\gamma^k\psi = -(e\hbar/2mi)[\bar{\psi}\partial_k\psi - (\partial_k\bar{\psi})\psi] - (e\hbar/2m)\partial_\nu(\bar{\psi}\sigma^{k\nu}\psi). \quad (20)$$

Here the first term is a convection current density associated with the translational motion of the electron. For an electron in a state with orbital angular momentum, this convection current density gives rise to an orbital magnetic moment. The second term is a spin current density, which is nonzero even in the rest frame of the electron.[14] For example, if the electron is in the state specified by Eq. (14), the flow lines for the spin current are closed circles, as they are for the momentum density, but of the opposite direction.[15] Obviously, such a current will generate a magnetic moment of the opposite direction as the spin.

To establish the general relationship between this magnetic moment and the spin, we decompose the spin current density into two terms:

$$j_S^k = -\frac{e\hbar}{2m}\partial_\nu(\bar{\psi}\sigma^{k\nu}\psi) = -\frac{e\hbar}{2m}\partial_n\bar{\psi}\sigma^{kn}\psi - \frac{e\hbar}{2mc}\frac{\partial}{\partial t}(\bar{\psi}\sigma^{k0}\psi). \quad (21)$$

This can be rewritten as

$$\mathbf{j}_S = \nabla\times\mathbf{M} + \frac{\partial\mathbf{P}}{\partial t}, \quad (22)$$

where

$$\mathbf{M} = -(e\hbar/2m)\psi^\dagger\gamma^0\boldsymbol{\sigma}\psi \quad (23)$$

and

$$\mathbf{P} = (ie\hbar/2mc)\psi^\dagger\gamma^0\boldsymbol{\alpha}\psi. \quad (24)$$

Thus $\mathbf{j}_S$ is the sum of a magnetization current density and a polarization current density. The former is associated with a magnetic moment per unit volume $\mathbf{M} = -(e\hbar/2m)\psi^\dagger\gamma^0\boldsymbol{\sigma}\psi$ and the latter with an electric dipole moment per unit volume $\mathbf{P} = (ie\hbar/2mc)\psi^\dagger\gamma^0\boldsymbol{\alpha}\psi$. Equation (23) implies that the magnetic moment of the electron is

$$\mathbf{m} = \int \mathbf{M}\,d^3x = -\frac{e\hbar}{2m}\int \psi^\dagger\gamma^0\boldsymbol{\sigma}\psi\,d^3x. \quad (25)$$

[Alternatively, the magnetic moment can be calculated as the moment of the magnetization current,

$$\mathbf{m} = \frac{1}{2}\int \mathbf{x}\times(\nabla\times\mathbf{M})d^3x. \quad (26)$$

An integration by parts shows that this expression is equivalent to Eq. (25).]

Comparing Eq. (25) with (18) we see that, apart from the factor of γ^0, the magnetic moment coincides with $-e/m$ times the spin. More precisely, the magnetic-moment operator coincides with $-e\gamma^0/m$ times the spin operator,

$$\mathbf{m}_{\text{op}} = -(e/m)\gamma^0\mathbf{S}_{\text{op}}. \quad (27)$$

This is, of course, the usual result for the magnetic moment of the electron. The standard derivation[16] of this result does not proceed via the definitions (25) or (26) of the magnetic moment; instead, it proceeds via the Dirac equation by investigating the response of the electron to an external magnetic field, a response that is found to have form expected for a magnetic moment. Thus the standard derivation fails to provide a physical picture of the mechanism underlying the magnetic moment. Incidentally, the standard derivation explicitly invokes the principle of minimal coupling. This principle enters the above calculation implicitly, through the assumption that the relevant current density is simply $-e\bar{\psi}\gamma^k\psi$, rather than some more complicated expression with, say, an extra term proportional to $\partial_\nu\bar{\psi}\sigma^{\mu\nu}\psi$ (such extra terms are required to account for the "anomalous" magnetic moments of the proton and the neutron).

Finally, what about the electric dipole moment, Eq. (24)? In the nonrelativistic limit, $\gamma^0\boldsymbol{\alpha}$ is an "odd" operator whose matrix elements are of order $1/m$. Hence $\mathbf{P}$ is of order $1/m^2$, which must be neglected in the nonrelativistic limit. This means that the electron has no electric dipole moment in its own rest frame. However, a moving electron has an electric dipole moment in the laboratory frame. This electric dipole moment can be regarded as arising from the relativistic transformation law for electromagnetic fields: A moving magnetic moment gives rise to an electric moment (and vice versa).

V. CONCLUSIONS

The calculations in the preceding sections should lay to rest the common misconception that spin is an essentially quantum-mechanical property. What these calculations show is that spin is essentially a wave property, but whether the wave is classical or quantum mechanical is of secondary importance. The only fundamental difference between the spins of a classical wave and a quantum-mechanical wave is that the spin of the former is a continuous macroscopic parameter, whereas the spin of the latter is quantized and is represented by a quantum-mechanical operator. The argument is often made that since the spin of a quantum-mechanical particle—such as photon—has a fixed magnitude, it is not possible to proceed to the classical limit of large quantum numbers, and consequently the spin must be regarded as a quantum property without classical analog. But this argument is flawed: Although we cannot proceed to the limit of large quantum numbers for a single particle, we can proceed to the limit of large occupation numbers for a system of many particles. A circularly polarized light wave is an example of a system in which the classical macroscopic spin angular momentum arises from the addition of a large number of quantum spins. Such a classical limit is also possible for electrons, but we must take the precaution of placing the electrons in different orbital states whenever we place them in the same spin state. The Einstein–de Haas effect and the magnetization found in permanent magnets involve classical limits brought about by a large number of electron spins and magnetic moments.

The physical picture of spin presented in the preceding sections has great intuitive appeal because it confirms our deep prejudice that angular momentum ought to be due to some kind of rotational motion. But the rotational motion consists of a circulation of energy in the wave fields, rather than a rotation of some kind of rigid body. The spin is *intrinsic*, or *inherent*, i.e., it is a fixed feature of the wave field that does not depend on environmental circumstances. But it is not *internal*, i.e., it is not within the internal structure of the electron or photon (of course, the structure of the wave field is crucial to the spin, but this is not what is usually meant by internal structure).

A conspicuous feature of the above physical picture is the close kinship of spin and orbital angular momentum: Both are due to the energy flow in the wave fields, and the distinction between them hinges on the mathematical separation of the angular momentum associated with the flow into two independent portions. Since this physical picture treats the spin and the orbital angular momentum in the same way, it gives us as good an understanding of spin as of orbital angular momentum. We no longer need to regard the spin as a mysterious entity.

[1]For the early history of spin, see the article by B. L. van der Waerden in *Theoretical Physics in the Twentieth Century*, edited by M. Fierz and V. F. Weisskopf (Interscience, New York, 1960); *Wolfgang Pauli: Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg u.a.*, edited by A. Hermann, K. V. Meyenn, and V. F. Weisskopf (Springer, New York, 1979); M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw-Hill, New York, 1966); and the articles by S. A. Goudsmit and G. E. Uhlenbeck in Phys. Today **29** (6), 40 (June, 1976).

[2]M. Jammer, Ref. 1, pp. 152 and 153.

[3]B. L. van der Waerden, Ref. 1, p. 216.

[4]For instance, P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford U. P., Oxford, 1958), p. 142; D. S. Saxon, *Elementary Quantum Mechanics* (Holden-Day, San Francisco, 1968), p. 191.

[5]A. Beiser, *Perspectives of Modern Physics* (McGraw-Hill, New York, 1969), p. 225, goes so far as to claim that "...Dirac was able to show on the basis of a relativistic quantum-mechanical treatment that particles having the charge and mass of the electron must have just the intrinsic angular momentum and magnetic moment attributed to them by Goudsmit and Uhlenbeck". This is somewhat of an exaggeration since, without prior knowledge of the spin of the electron, we cannot know that Dirac's equation is applicable.

[6]F. J. Belinfante, Physica **6**, 887 (1939).

[7]W. Gordon, Z. Phys. **50**, 630 (1928).

[8]For example, G. Wentzel, *Quantum Theory of Fields* (Interscience, New York, 1949).

[9]For instance, D. Hestenes, Am. J. Phys. **47**, 5 (1979).

[10]A clear discussion of the canonical versus the symmetrized energy-momentum tensor is given by D. E. Soper, *Classical Field Theory* (Wiley, New York, 1976).

[11]L. Rosenfeld, Mem. Acad. R. Belg. **18**, no. 6 (1940).

[12]The notation for spinors employed here is that of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964). The notation of Wentzel is slightly different.

[13]The Gordon decomposition is often used in spinor calculations (see, e.g., Ref. 12), but its importance in establishing a physical picture for the origin of spin seems to have been forgotten.

[14]Note that the convection current and the spin current are separately conserved:

$$\partial_\mu[\bar{\psi}\,\partial^\mu\psi - (\partial^\mu\bar{\psi})\psi] = 0 \text{ and } \partial_\mu\,\partial_\nu(\bar{\psi}\sigma^{\mu\nu}\psi) = 0.$$

This is an immediate consequence of the antisymmetry of $\sigma^{\mu\nu}$.

[15]Within the nonrelativistic approximation, the "small" components can be ignored when evaluating the right-hand side of Eq. (20), but they cannot be ignored when evaluating the left-hand side.

[16]Reference 12.

The quest for ultrahigh energies

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A categorization is given of all the methods for accelerating particles. It is shown that in principle one can employ the large fields of a laser for this purpose as well as the wake fields of intense low-energy particle beams. Discussion is given of four acceleration schemes which offer the possibility of attaining very high-energy particles; namely, the inverse free-electron laser accelerator, the beat-wave accelerator, the wake-field accelerator, and the two-beam accelerator.

I. INTRODUCTION

Ever since Cockcroft and Walton first produced nuclear reactions by means of a particle accelerator, in that case an electrostatic accelerator, physicists have bent their ingenuity to the development of ever-more powerful machines. The devices which have been developed include some remarkable machines, such as the cyclotron and the betatron, and some truly innovative concepts such as strong focusing and stochastic cooling.[1–4]

Of course, the driving force behind this effort has been the ever-opening science which ever-higher energy has made possible. The machines on the forefront of elementary particle physics are truly marvels of engineering. One thinks of the Tevatron at Fermilab or the CERN Super Proton Synchrotron (SPS), with which the intermediate bosons were discovered in 1983. Today, the physics of elementary particles demands very *large* machines such as these two, and under construction are even larger machines such as the Large Electron Positron Collider (LEP) which will have a circumference of 27 km. Under serious consideration is the Superconducting Super Collider (SSC), the arguments for which have been presented recently.[5]

Although the arguments for the SSC are most compelling, and we believe that it should be built, it is clear that the progression of ever-larger machines cannot go on forever. Yet, one can be sure that the scientific desire for ever-higher energies will continue unabated. In fact, if one looks back over the last five decades, then one sees an almost exponential rise in the available particle energy, as is de-

